Availability, MTTF and Cost analysis of complex system under Preemptive resume repair policy using copula distribution

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Abstract
In the present paper authors have focused on the study of complex system consisting two subsystems, in series configuration and handling by a human operator. The subsystem (1) has three units at super priority, priority and ordinary & the subsystem (2) has one unit in series configuration with the subsystem-1. The whole system is operated by a human operator and human failure can also appear at different state where system is in operational mode. Initially super priority unit starts function and failure during of super priority unit the priority unit start functioning and super priority unit goes under repair. The primitive resume repair policy is employed for repair of subsystem-1. The all failure rates are assumed to constants and follow exponential distribution but repair follow general and Gumbel-Hougaard family copula distribution. The system is studied by supplementary variable technique and Laplace transform. Various measure of reliability such as availability, state transition probabilities, mean time to system failure(M.T.T.F) and profit function has been discussed for available maintenance cost for all time and profit incurred by unit time for given interval. Some particular cases have been discussed for different values of different rates.

Keywords: Reliability, Availability. Human failure, Preemptive resume repair policy, super priority, M.T.T.F. and profit function.

I. Introduction
Many researcher whose works referred as Alistair (2007), Cox (1995), Dilip (2014) and Oliverira et al. (2005) studied repairable complex system and proclaimed their validation in the field of reliability by taking different failure rates and common assumptions that one type of (general) repair is possible between two transition state. Thus whenever the system fails, one type of repair is employed to repair the system which takes more times for repair of failed unit, resulting the industry suffers with a great loss. Usually the researchers consider that only one repair that is possible between two adjacent states i.e. failed state and operational state. Many authors including Govil, A. (1974), Ram. M & Singh (2008 &2010), and Cai. X et al. (2005) studied the complex systems under different type of failure and preemptive resume repair policy. The researchers Dhillon et al. (1992 & 1993), Vanderperre. E. J. (1990) & Ram. M and.et al. (2013) studied reliability characteristic for a complex system with common cause failure and reliability of duplex standby system by suplimentary variable technique (Cox, 1995) and Laplace transform. But there are many situations in real life where more than one repair is require between two transition states for quick maintenance of failed unit. When such type of situation arises, the system is studies by using Copula (Gennheimer. H, 2002 and Nelson,
2006). The authors (Singh et al. 2010) studied a complex system which have three units superpriority, priority and ordinary under preemptive resume repair policy using different types of failure and repair by using Gumbel-Hougaard family Copula distribution. If the system is running under minor partial failure and the system is in operation mode general repair can be employed but whenever the system is in complete failure mode the system is to be repaired using Copula [Gumbel-Hougaard family copula] distribution. Refered to this strategy Ghasemi. A et al. (2010) and Ram et al (2013) studies the complex system having two subsystem with controller and standby complex system with waiting repair policy using Gumbel-Hougaard family copula distribution. The prime aim in any industry or organization is to gain more profit with a least expenditure. There for the minor partially failed state when a system is in working with degraded mode, the general repair is enough but if system is in completely failure mode the copula distribution should be employed for quick maintenance of the system. Here in the system the authors have considered a complex system which consists of two subsystem, subsystem-1 & subsystem-2. The subsystem-1 has three units super priority, priority and ordinary units & is working under preemptive resume repair policy & the subsystem-2 has one unit attached with subsystem-1 in series configuration. The system is operated by a human operator. Initially the super priority unit of subsystem-1 starts working and other units-priority and ordinary unit remains in warm standby mode. When super priority unit fails the priority unit starts of working and the failed unit gets for repair. If super priority unit repair before failure of priority unit then super priority unit starts functioning and the priority unit will go for standby mode. In case priority unit fail before repair of super priority unit then ordinary unit start functioning and priority unit will have to wait for repair. As soon as super priority unit is repaired, it start function and priority unit will go for repair and after get repair it goes for standby mode. Thus super priority unit will never be in standby mode. The system is operated by human operator; the human error can arise at any stage when the system is in operational mode. The system will completely fail in following situation: 1) ordinary unit of subsystem-1 fails before the repair of the super priority unit. 2) subsystem-2 fails whenever subsystem-1 is in operational mode. 3) Human failure can appear at any stage when subsystem-1 is in operational mode.

All failure rates are constants and follows exponential distribution, however repair follows two types of distribution namely: General and Gumbel-Hougaard family copula distribution. Whenever the system is in partially failed state [S₀, S₁, S₂, S₃, S₅, S₆, S₇] the general repair is employed but whenever the system is in completely failed state [S₄, S₈, S₉], it is repaired by Gumbel-Hougaard family copula distribution. The system is studied by supplementary variable technique & Laplace Transform. Various measures of reliability such as availability, state transition probability, mean time to system failure MTSF profit function have been discussed. Some particular cases also have been highlighted for different value of failure rates. The results are demonstrated by graphs and conclusion have been drawn.
II. State description

<table>
<thead>
<tr>
<th>State</th>
<th>State Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>The state indicates that the super priority unit has been failed, priority unit start functioning and superpriority unit is under repair.</td>
</tr>
<tr>
<td>S₂</td>
<td>The priority unit has also been failed and supper priority unit is running under repair, system is in operational mode.</td>
</tr>
<tr>
<td>S₃</td>
<td>The super priority unit has been repaired and is in operational mode, priority unit is running under repair.</td>
</tr>
<tr>
<td>S₄</td>
<td>The system has completely failed due to failure of subsystem-1.</td>
</tr>
<tr>
<td>S₅</td>
<td>In this state the super priority unit has start functioning after repaired and priority unit is running repair.</td>
</tr>
<tr>
<td>S₆</td>
<td>The super priority unit is in operational mode, priority unit is in standby mode and ordinary unit in running under repair.</td>
</tr>
<tr>
<td>S₇</td>
<td>In state S₇ the superpriority unit of subsystem-1 has fail and priority unit is in operational mode, the system is in operational mode.</td>
</tr>
<tr>
<td>S₈</td>
<td>In this state the system is in a completely failed due to failure of subsystem-2.</td>
</tr>
<tr>
<td>S₉</td>
<td>The system has completely failed due to human failure of operator.</td>
</tr>
</tbody>
</table>

The state description highlight that S₀ is a state where the system is in perfect state where both subsystems are in good working condition. S₁, S₂, S₃, S₅, S₆, S₇ are the states where the system is in degraded mode and the repair is being employed, states S₄, S₈, and S₉ are the states where the system is in completely failure mode.

III. Assumption

The following assumptions are taken throughout the discussion of the model:

(i) Initially the system is in S₀ state and all units of subsystem-1 and subsystem-2 are in good working condition.

(ii) The subsystem-1 works successfully till ordinary unit is in good working condition.

(iii) Subsystem-1 fails if ordinary unit fail before repair of superpriority unit.

(iv) The units of subsystem-1 are in warm stated by mode and ready to start within a negligible period of time after failure of any unit of subsystem-1.

(v) The system can be repaired when it is in degraded state or completely failed state.

(vi) All failure rates are constant and they follow an exponential distribution.

(vii) Human failure /complete failure system needs immediate repairing (by Gumbel-Hougaard family copula).

(viii) A repaired system works like a new system and there will be no damage done due to repair.

(ix) As soon as the failed unit gets repair it ready to perform the task.
IV. Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Time variable on time scale.</td>
</tr>
<tr>
<td>$S$</td>
<td>Laplace transform variable.</td>
</tr>
<tr>
<td>$\lambda_h/\lambda_B$</td>
<td>Failure rates of due to human failure/ failure rate of subsystem-2.</td>
</tr>
<tr>
<td>$\lambda_s/\lambda_p/\lambda_o$</td>
<td>Failure rates of superpriority/ priority/ and ordinary units of subsystem-1.</td>
</tr>
<tr>
<td>$\eta(x)/\phi(y)/\psi(x)$</td>
<td>Repair rate for superpriority/ priority/ and ordinary unit.</td>
</tr>
<tr>
<td>$P_i(t)$</td>
<td>The probability that the system is in $S_i$ state at instant ‘t’ for $i=0$ to 9.</td>
</tr>
<tr>
<td>$\Phi(s)$</td>
<td>Laplace transformation of $P(t)$.</td>
</tr>
<tr>
<td>$P_j(x, t)$</td>
<td>The probability that a system is in state $S_o$ for $j=1$ to 9; the system is running under repair and elapsed repair time is $x, t$.</td>
</tr>
<tr>
<td>$E_p(t)$</td>
<td>Expected profit during the interval $[0, t)$.</td>
</tr>
<tr>
<td>$K_1, K_2$</td>
<td>Revenue and service cost per unit time respectively.</td>
</tr>
<tr>
<td>$\mu_0(x)=C_0(u_1(x), u_2(x))$</td>
<td>The expression of joint probability (failed state $S_i$ to good state $S_0$) according to Gumbel-Hougaard family copula is given as $C_\theta(u_1(x), u_2(x)) = \exp[\alpha + \log(\phi(x))]^{\alpha}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where $\theta$ is a parameter.</td>
</tr>
</tbody>
</table>

Fig.1 System Configuration

Fig.2 State Transition Diagram
V. Formulation and solution of Mathematical Model

By the probability of considerations and continuity arguments, the following set of difference differential equations are associated by the present mathematical model.

\[
\begin{align*}
\frac{\partial}{\partial t} + \lambda_s + \lambda_B + \lambda_n \left[ P_0(t) = \int_0^\infty \eta(x)P_1(x,t)dx + \int_0^\infty \mu_0(x)P_6(x,t)dx + \int_0^\infty \phi(z)P_5(z,t)dz \\
+ \int_0^\infty \phi(y)P_3(y,t)dy + \int_0^\infty \mu_0(x)P_0(x,t)dx
\end{align*}
\] (1)

\[
\begin{align*}
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_n + \eta(x) \left[ P_1(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_n + \eta(x) \left[ P_2(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_n + \eta(x) \left[ P_3(y,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_n + \eta(x) \left[ P_4(z,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_n + \eta(x) \left[ P_5(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \left[ P_6(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \left[ P_7(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \left[ P_8(x,t) = 0 \right. \\
\left. \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \left[ P_9(x,t) = 0 \right.
\end{align*}
\] (2)

Boundary conditions

\[
P_1(0,t) = \lambda_s P_0(t) \] (11)

\[
P_2(0,t) = \lambda_p P_1(0,t) + \lambda_p P_3(0,t) \] (12)

\[
P_3(0,t) = \int_0^\infty \eta(x)P_2(x,t)dx \] (13)

\[
P_4(0,t) = \lambda_0 P_2(0,t) + \lambda_0 P_3(0,t) + \lambda_p P_1(0,t) \] (14)
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\[ P_5(0, t) = \int_0^\infty \eta(x)P_4(x, t)dx \] \hspace{1cm} (15)

\[ P_6(0, t) = \int_0^\infty \eta(x)P_7(x, t)dx + \int_0^\infty \phi(y)P_5(y, t)dy \] \hspace{1cm} (16)

\[ P_7(0, t) = \lambda_s P_6(0, t) \] \hspace{1cm} (17)

\[ P_8(0, t) = \lambda_B (P_0(0, t) + P_1(0, t) + P_2(0, t) + P_3(0, t) + P_5(0, t) + P_6(0, t) + P_7(0, t)) \] \hspace{1cm} (18)

\[ P_9(0, t) = \lambda_h (P_0(0, t) + P_1(0, t) + P_2(0, t) + P_3(0, t) + P_5(0, t) + P_6(0, t) + P_7(0, t)) \] \hspace{1cm} (19)

Solution of the Model

Taking Laplace transformation of equations (1)-(19) and using equation with help of initial condition, \( P_0(0)=1 \), one can obtain.

\[ [s + \lambda_s + \lambda_B + \lambda_h + \eta(x)] \tilde{P}_0(s) = 1 + \int_0^\infty \eta(x)\tilde{P}_1(x, s)dx + \int_0^\infty \mu_0(x)\tilde{P}_2(x, s)dx + \int_0^\infty \mu_0(x)\tilde{P}_4(x, s)dx + \int_0^\infty \mu_0(x)\tilde{P}_5(x, s)dx + \int_0^\infty \mu_0(x)\tilde{P}_6(x, s)dx + \int_0^\infty \phi(y)\tilde{P}_7(y, s)dy + \int_0^\infty \phi(z)\tilde{P}_8(z, s)dx \] \hspace{1cm} (20)

\[ [s + \frac{\partial}{\partial x} + \lambda_s + \lambda_B + \lambda_h + \eta(x)] \tilde{P}_1(x, s) = 0 \] \hspace{1cm} (21)

\[ [s + \frac{\partial}{\partial x} + \lambda_o + \lambda_B + \lambda_h + \eta(x)] \tilde{P}_2(x, s) = 0 \] \hspace{1cm} (22)

\[ [s + \frac{\partial}{\partial y} + \lambda_s + \lambda_B + \lambda_h + \phi(y)] \tilde{P}_3(y, s) = 0 \] \hspace{1cm} (23)

\[ [s + \frac{\partial}{\partial x} + \eta(x)] \tilde{P}_4(x, s) = 0 \] \hspace{1cm} (24)

\[ [s + \frac{\partial}{\partial y} + \lambda_s + \lambda_B + \lambda_h + \phi(y)] \tilde{P}_5(y, s) = 0 \] \hspace{1cm} (25)

\[ [s + \frac{\partial}{\partial z} + \lambda_s + \lambda_B + \lambda_h + \phi(z)] \tilde{P}_6(z, s) = 0 \] \hspace{1cm} (26)

\[ [s + \frac{\partial}{\partial x} + \lambda_o + \lambda_B + \lambda_h + \eta(x)] \tilde{P}_7(x, s) = 0 \] \hspace{1cm} (27)

\[ [s + \frac{\partial}{\partial x} + \mu_0(x)] \tilde{P}_8(x, s) = 0 \] \hspace{1cm} (28)
\[
\begin{align*}
&\left[ s + \frac{\partial}{\partial x} + \mu_0(x) \right] \overline{P}_6(x,s) = 0 \quad (29)
\end{align*}
\]

Laplace transform of boundary conditions

\[
\begin{align*}
&\overline{P}_1(0,s) = \lambda_s \overline{P}_0(s) \quad (30) \\
&\overline{P}_2(0,s) = \lambda_p \overline{P}_1(0,s) + \lambda_s \overline{P}_3(0,s) \quad (31) \\
&\overline{P}_3(0,s) = \int_0^\infty \eta(x) \overline{P}_2(x,s) \, dx \quad (32) \\
&\overline{P}_4(0,s) = \lambda_p \overline{P}_1(0,s) + \lambda_s \overline{P}_5(0,s) + \lambda_o \overline{P}_2(0,s) \quad (33) \\
&\overline{P}_5(0,s) = \int_0^\infty \eta(x) \overline{P}_4(x,s) \, dx \quad (34) \\
&\overline{P}_6(0,s) = \int_0^\infty \phi(y) \overline{P}_5(y,s) \, dy + \int_0^\infty \eta(x) \overline{P}_7(y,s) \, dx \quad (35) \\
&\overline{P}_7(0,s) = \lambda_s \left[ \int_0^\infty \phi(y) \overline{P}_5(y,s) \, dy + \int_0^\infty \eta(x) \overline{P}_7(y,s) \, dx \right] \quad (36) \\
&\overline{P}_8(0,s) = \lambda_B \overline{P}_0(s) + \overline{P}_1(0,s) + \overline{P}_2(0,s) + \overline{P}_3(0,s) + \overline{P}_5(0,s) + \overline{P}_7(0,s) + \overline{P}_6(0,s) \quad (37) \\
&\overline{P}_9(0,s) = \lambda_g (\overline{P}_0(s) + \overline{P}_1(0,s) + \overline{P}_2(0,s) + \overline{P}_3(0,s) + \overline{P}_5(0,s) + \overline{P}_7(0,s) + \overline{P}_6(0,s)) \quad (38)
\end{align*}
\]

Solving (21)-(29), with help of equations (30) to (38) one may get,

\[
\begin{align*}
&\overline{P}_0(s) = \frac{1}{D(s)} \quad (39) \\
&\overline{P}_1(s) = \frac{\lambda_s}{D(s)} \frac{(1 - S_g(s + \lambda_B + \lambda_o + \lambda_h))}{(s + \lambda_B + \lambda_o + \lambda_h)} \quad (40) \\
&\overline{P}_2(s) = \frac{\lambda_s}{D(s)A} \frac{(1 - S_g(s + \lambda_B + \lambda_o + \lambda_h))}{(s + \lambda_B + \lambda_o + \lambda_h)} \quad (41) \\
&\overline{P}_3(s) = \frac{\lambda_s \lambda_p S_g(s + \lambda_B + \lambda_o + \lambda_h)(1 - S_g(s + \lambda_B + \lambda_o + \lambda_h))}{D(s)A} \quad (42) \\
&\overline{P}_4(s) = \frac{\lambda_o \lambda_p \lambda_s H}{D(s)(F,H - C)A} \frac{(1 - S_g(s))}{(s)} \quad (43) \\
&\overline{P}_5(s) = \frac{\lambda_o \lambda_p \lambda_s H}{D(s)(F,H - C)A} \frac{(1 - S_g(s + \lambda_B + \lambda_o + \lambda_h))}{(s + \lambda_B + \lambda_o + \lambda_h)} \quad (44)
\end{align*}
\]
\[ P_o(s) = \frac{\lambda_s C}{D(s)(F.H - C)} A \left(1 - S_\phi (s + \lambda_B + \lambda_s + \lambda_h)\right) \]  
(45)

\[ P_7(s) = \frac{\lambda_o \lambda_s C}{D(s)(F.H - C)} A \left(1 - S_\eta (s + \lambda_B + \lambda_p + \lambda_h)\right) \]  
(46)

\[ P_8(s) = P_7(0, s) \frac{(1 - S_{\mu_0}(s))}{(s)} \]  
(47)

\[ P_9(s) = P_7(0, s) \frac{(1 - S_{\mu_0}(s))}{(s)} \]  
(48)

\[ D(s) = s + \lambda_s + \lambda_B + \lambda_h - (1 - H) - (1 + \lambda_s)(\lambda_B + \lambda_h) S\mu_0(s) \]

\[ \begin{array}{l}
\left\{ \lambda B \lambda_o \lambda_s S_\eta (s) + \lambda_B \lambda_s C + \lambda_B \lambda_o \lambda_s C + \right.
\lambda_o \lambda_p \lambda_s H S_\eta (s) + \lambda_o \lambda_s C + \lambda_h \lambda_o \lambda_s C
\end{array} \quad \frac{S\mu_0(s)}{(F.H - C)A} - \]

\[ \begin{array}{l}
\lambda_q \lambda_p \lambda_s \lambda_h \lambda_s \eta (s + \lambda_o + \lambda_h) \quad \quad \left(1 + S_\eta (s + \lambda_B + \lambda_o + \lambda_h)\right) S\mu_0(s) -
\end{array} \]

\[ \begin{array}{l}
\left\{ \lambda_q C \quad \frac{(F.H - C)}{A} + \lambda_p \lambda_s \lambda_o \lambda_h \right. \end{array} \left(1 + S_\eta (s + \lambda_B + \lambda_o + \lambda_h)\right) \]

\[ \frac{S\phi (s + \lambda_B + \lambda_s + \lambda_h)}{A} \]

Where,  
\[ A = (1 - \lambda_s S_\eta (s + \lambda_B + \lambda_o + \lambda_h)) \],  
\[ F = (1 - \lambda_s S_\phi (s)) \],  
\[ C = \lambda_p \lambda_s S_\eta (s) S\phi (s + \lambda_B + \lambda_s + \lambda_h) \],  
\[ H = (1 - \lambda_s S_\eta (s + \lambda_B + \lambda_p + \lambda_h)) \]

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

\[ P_{up}(s) = P_0(s) + P_1(s) + P_3(s) + P_5(s) + P_6(s) + P_7(s) \]

\[ = \frac{1}{D(s)} \left[ 1 + \frac{\lambda_s (1 - S_\phi (s + \lambda_B + \lambda_o + \lambda_h))}{(s + \lambda_B + \lambda_o + \lambda_h)} + \frac{\lambda_s (1 - S_\eta (s + \lambda_B + \lambda_o + \lambda_h))}{(s + \lambda_B + \lambda_o + \lambda_h)} + \right. \]

\[ \frac{\lambda_s \lambda_p S_\eta (s + \lambda_B + \lambda_o + \lambda_h)}{(s + \lambda_B + \lambda_o + \lambda_h)} \left(1 - S_\eta (s + \lambda_B + \lambda_o + \lambda_h)\right) + \]

\[ \frac{\lambda_s \lambda_p \lambda_s H}{(F.H - C)A} + \frac{\lambda_s C}{(F.H - C)A} \right. \left(1 - S_\phi (s + \lambda_B + \lambda_p + \lambda_h)\right) \]

\[ \frac{\lambda_s \lambda_p \lambda_s H}{(F.H - C)A} + \frac{\lambda_s \lambda_p \lambda_s H}{(F.H - C)A} \]

\[ \left(1 - S_\phi (s + \lambda_B + \lambda_s + \lambda_h)\right) \]  
(49)

\[ \bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \]  
(50)

Particular Cases
**A. Availability**

When repair follows exponential distribution, setting

$$S_{p_0}(s) = \frac{\exp[\theta(x)]^\theta}{s + \exp[\theta(x)]^\theta} = \frac{\exp[x^\theta + \{\log \phi(x)\}^\theta]}{s + \exp[x^\theta + \{\log \phi(x)\}^\theta]}.$$

$$S_{\phi}(s) = \frac{\phi}{s + \phi}, \quad S_{\eta}(s) = \frac{\eta}{s + \eta},$$

and taking the values of different parameters as $\lambda_s=0.01, \lambda_B=0.02, \lambda_p=0.03, \lambda_o=0.04, \phi=1, \theta=1, x=1$, in (49), then taking the inverse Laplace transform, one can obtain,

$$P_{up}(t) = 0.016357e^{-2.768484t} - 0.001031e^{-2.718009t} - 0.00253e^{-2.217303t} -$$

$$(0.000571\cos(0.010681t) + 0.002655\sin(0.010681t))e^{-1.077369t}$$

$$+ 0.982709e^{-0.009565} + (0.003458\cos(0.001951t) - 0.001781\sin(0.001951t))e^{-1.062871t} - 0.005004e^{-1.049877t} + 0.001296$$

(51)

For, $t=0, 2, 4, 6, 8, 10, 12, 14, 16, 18$; units of time, one may get different values of $P_{up}(t)$ with the help of (51) as shown in Fig.3 calculated.

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.9816</td>
</tr>
<tr>
<td>4</td>
<td>0.9801</td>
</tr>
<tr>
<td>6</td>
<td>0.9794</td>
</tr>
<tr>
<td>8</td>
<td>0.9782</td>
</tr>
<tr>
<td>10</td>
<td>0.9771</td>
</tr>
<tr>
<td>12</td>
<td>0.9761</td>
</tr>
<tr>
<td>14</td>
<td>0.9750</td>
</tr>
<tr>
<td>16</td>
<td>0.9739</td>
</tr>
<tr>
<td>18</td>
<td>0.9728</td>
</tr>
</tbody>
</table>

Fig. 3: Availability as function of time

**B. Mean Time to Failure (M.T.T.F.)**

Taking all repairs zero and the limit as $s$ tends to zero in (49) for the exponential distribution, one can obtain the expression for M.T.T.F. as:

$$M.T.T.F. = \lim_{s \to 0} \frac{1}{\lambda_s + \lambda_B + \lambda_h} \left(1 + \frac{\lambda_s (1 + \lambda_p)}{\lambda_p + \lambda_B + \lambda_h}\right).$$

(52)

Setting $\lambda_B=0.02, \lambda_h=0.03, \lambda_p=0.015$ and varying $\lambda_s, \lambda_p, \lambda_h, \lambda_B$ one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (52), one may obtain the variation of M.T.T.F. with respect to failure rates as shown in Fig.4.
C. Cost Analysis

If the service facility be always available, then expected profit during the interval \([0, t)\) is

\[
E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t \tag{53}
\]

For the same set of parameter of (49), one can obtain (53).

Therefore

\[
E_p(t) = K_1 (-0.006023e^{-2.768488} + 0.000233e^{-2.718000} - 0.000325e^{-2.217303} + \\
(0.000554 \cos(0.010681t) + 0.002459 \sin(0.010681t))e^{-1.077369} \\
- 1737.876752e^{-0.000563} + (0.003251 \cos(0.001951t) - 0.001682 \\
\sin(0.001951t))e^{-1.062871} + 0.004767e^{-1.049877} - 0.001310e^{-0.988884} \\
+ 1737.881835) - K_2 t \tag{54}
\]

Setting \(K_1 = 1\) and \(K_2 = 0.5, 0.25, 0.15, 0.10\) and 0.05 respectively and varying \(t = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\) units of time, the results for expected profit can be obtain as shown in Fig.5.
**VI. Conclusions**

Fig. 3 provides information how the availability of the complex repairable system changes with respect to the time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_s = 0.01$, $\lambda_p = 0.015$, $\lambda_o = 0.04$, $\lambda_B = 0.02$, $\lambda_h = 0.03$, availability of the system decreases with a very high up to time $t=2$ but afterward the variation becomes slowly and probability of failure increases, with the passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Fig. 4, yields the mean-time-to-failure (M.T.T.F.) of the system with respect to variation in $\lambda_s$, $\lambda_p$, $\lambda_h$, and $\lambda_B$ respectively when the other parameters have been taken as constant. The variation in MTTF corresponding to failure rates $\lambda_s$, $\lambda_p$ are almost is very closure but corresponding to $\lambda_B$, $\lambda_h$ the variation is very high which indicates that these both are more responsible to proper operation of the system.

When revenue cost per unit time $K_1$ is fixed at 1, service costs $K_2 = 0.5$, 0.25, 0.15, 0.10, 0.05, profit has been calculated and results are demonstrated by graphs in Fig.5. A critical examination from Fig.5 reveals that expected profit increases with respect to the time when the service cost $K_2$ fixed at minimum value 0.05. Finally, one can observe that as service cost increase, profit decrease. In general for low service cost, the expected profit is high in comparison to high service cost.
References


Appendix

Appendix 1: For study the particular cases in solution part of paper the authors have use the maple 7 software for computation purpose such as taking inverse Laplace transform and computing the availability and MTTF and profit from the expression in equation 51, 52 & 54.

Appendix 2: The system to be in state $S_i$ during time $t$ and $t+\Delta t$ is that the system should not move to any other state i.e the probability that the system will be in state $i=0$ (for illustration) say $S_0$ is;

$$P_i(t + \Delta t) = (1 - \lambda_i \Delta t)(1 - \lambda_{si} \Delta t)(1 - \lambda_{ai} \Delta t) P_i(t) + \int_0^\infty \eta(x) P_i(x, t) \Delta t dx + \int_0^\infty \mu_i(x) P_i(x, t) \Delta t dx + \int_0^\infty \phi(z) P_i(z, t) \Delta t dz$$

$$+ \int_0^\infty \phi(y) P_i(y, t) \Delta t dy + \int_0^\infty \mu_i(x) P_i(x, t) \Delta t dx$$

$$Lt \to 0 \frac{P_i(t + \Delta t) - P_i(t)}{\Delta t} + (\lambda_i + \lambda_{si} + \lambda_{ai}) P_i(t) + (\Delta t + (\Delta t)^2 + \ldots) P_i(t) = \int_0^\infty \eta(x) P_i(x, t) dx + \int_0^\infty \mu_i(x) P_i(x, t) dx +$$

$$\int_0^\infty \phi(z) P_i(z, t) dz + \int_0^\infty \phi(y) P_i(y, t) dy + \int_0^\infty \mu_i(x) P_i(x, t) dz$$

OR Equation 1;

$$\left[ \frac{\partial}{\partial t} + \lambda_i + \lambda_{si} + \lambda_{ai} \right] P_i(t) = \int_0^\infty \eta(x) P_i(x, t) dx + \int_0^\infty \mu_i(x) P_i(x, t) dx + \int_0^\infty \phi(z) P_i(z, t) dz +$$

$$\int_0^\infty \phi(y) P_i(y, t) dy + \int_0^\infty \mu_i(x) P_i(x, t) dz$$

(1)